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Connective constant of SAWs on the Sierpinski gasket family

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Abstract. Using a graph counting technique suitable for regular fractals, an exact evaluation of the total number of embeddings of self-avoiding walks on the generalized Sierpinski gasket is obtained. Numerical estimates for the connective constants μ_b are quoted for the first time, where b is the generation parameter of the gaskets. It is shown that the number of distinct n-step SAWs per site c_n converges to the triangular lattice values when $b \to \infty$ ($D_f \to 2$). Our analysis indicates that μ_b converges to the Euclidean value in the same limit and an asymptotic expression is given.

1. Introduction

The statistics of self-avoiding walks (SAWs) on fractals has been extensively investigated using exact renormalization techniques [1–3], by finite-size scaling arguments [4], Monte Carlo techniques [5] and, more recently, with series expansions [6]. This problem is relevant to the study of polymers in a dilute solution confined to a highly disordered media.

These studies show that the critical properties of SAWs on fractals depend on several geometrical parameters besides the Hausdorff dimension D_f. A question that naturally arises is the convergence of critical exponents to Euclidean lattices values when D_f approaches an integer dimension. This work adresses this problem, which has been intensively investigated in the last few years [5, 7–9].

We study SAWs on a family of finitely ramified regular fractals embedded in the two-dimensional Euclidean space, the generalized Sierpinski gasket. Each member of this family is constructed from a generator characterized by a parameter b, where b is an integer which runs from two to infinity. Each generator is an equilateral triangle (see figure 1) containing b^2 smaller triangles from which the downward-oriented ones are discarded and the $b(b+1)/2$ upward-oriented triangles are left. The corresponding regular fractal is formed by reproducing iteratively the generator in each upward-oriented triangle. The lattice at an s-stage of construction (after s iterations) is then obtained from the generator with all upward-oriented smaller triangles filled with the reproductions of the previous $(s-1)$-stage. The lattice at the first stage is the generator.

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In this treatment, the sequence of coefficients $c_n(b)$ are considered for each value of b independently. We then proceed with a new analysis and consider $c_n(b)$ as a sequence of functions (with index n) of the continuous parameter b. The asymptotic behaviour of $c_n(b)$ when $b \to \infty$ for each value of n is obtained and it is shown analytically that $\lim_{b\to\infty} c_n(b) = c_n(T)$, that is, each coefficient converges asymptotically to the triangular value.

Our analysis of the coefficients $c_n(b)$ indicate that $\lim_{b\to\infty} \mu_b = \mu_T$ with the first-order correction term of the order $\frac{1}{b^2}$.

The numerical estimates of μ_b confirm the above result. This is the first numerical evidence of the convergence of a critical parameter of SAWs to the corresponding Euclidean value when $b \to \infty$ in the family of generalized Sierpinski gaskets.

Using an exact enumeration technique to calculate $c_n(b)$ we obtain series expansions that are exact order by order. The results presented here for μ_b have good accuracy and can be systematically improved by enlarging the order of the series.

The investigation of the statistic of SAWsz on fractals based on the series-expansion method would also be helpful to settle some open question and conjectures regarding the limiting behaviour of the associated critical exponents as $b \to \infty$. Work along these lines is in progress.

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